

# $U_A(1)$ Anomaly in Background Fields Dominated by QCD-Monopoles on $SU(2)$ Lattice

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## Abstract

We study  $U_A(1)$  anomaly of non-perturbative QCD in the maximally abelian gauge on  $SU(2)$  lattice. The existence of the strong correlation between QCD-monopoles and instantons in the abelian gauge is shown by both analytic and numerical works including lattice simulations. These results bring us a conjecture that the  $U_A(1)$  symmetry would be explicitly broken in the background fields dominated by QCD-monopoles. We find an evidence for our conjecture by measuring the chiral-asymmetric zero modes of the Dirac operator in the backgrounds of QCD-monopoles.

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As for the appearance of color-magnetic monopoles in  $SU(N_c)$  gauge theory, 't Hooft proposed an interesting idea of the abelian gauge fixing [1], which is defined by the gauge transformation in the coset space of the gauge group in order to fix the gauge degrees of freedom up to the maximally abelian subgroup. In the abelian gauge, point-like singularities in the three-dimensional space  $\mathbf{R}^3$  under the maximally abelian subgroup can be identified as color-magnetic monopoles [1], which will be called QCD-monopoles hereafter. In other words, QCD-monopoles originate from the same topological nature as the 't Hooft-Polyakov monopoles, which correspond to the homotopy group  $\pi_2(SU(N_c)/U(1)^{N_c-1}) = Z_\infty^{N_c-1}$  [2]. However, QCD-monopoles have no condition to exist as classically stable and/or finite-energy solutions, unlike the 't Hooft-Polyakov monopoles are the explicit solutions of the field equation [2]. One thus conjectures that the realistic model of QCD vacuum would be characterized by the highly quantum feature of QCD-monopoles, *e.g.* its condensation, rather than the classical one [3]. Then the dual Meissner effect, which yields the exclusion of the color-electric fields, must be realized [3]. The recent lattice QCD simulations [4]–[6] support this conjecture that QCD-monopoles play a crucial role on color confinement through their condensation<sup>†</sup>.

As well known, QCD has also classical and non-trivial gauge configurations, *i.e.* instantons as topological defects in the Euclidean space  $\mathbf{R}^4$  corresponding to the homotopy group  $\pi_3(SU(N_c)) = Z_\infty$  [2]. It seems that instantons and QCD-monopoles are thought to be hardly related to each other since these topological objects appear from different non-trivial homotopy groups. However, the recent analytical works have demonstrated the QCD-monopole as a classically stable solution in the background fields of the instanton configuration using the abelian gauge fixing [7,8,10–12]. Furthermore, the several lattice QCD simulations have shown the existence of the strong correlation between instantons and QCD-monopoles in

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<sup>†</sup>QCD-monopole condensation is characterized by the presence of the long and tangled monopole trajectories in the four-dimensional space  $\mathbf{R}^4$  and can be interpreted as the Kosterlitz-Thouless type phase transition [6].

the highly quantum vacuum [9,10,13,14] as well as the semi-classical vacuum [12,15,16].

Here, we remind that instantons are important topological objects in QCD relating to the  $U_A(1)$  problem [17]. It is well known that the index of the massless Dirac operator in the instanton background fields is equal to the Pontryagin index, *i.e.* the topological charge  $Q$ :

$$n_+ - n_- = Q \quad , \quad (1)$$

where  $n_+$  ( $n_-$ ) is the number of zero modes with the positive (negative) chirality. The previous relation is well known as the Atiyah-Singer index theorem. The index of the Dirac operator corresponds to the number of chiral-asymmetric zero modes that yield the chiral anomaly in the global  $U_A(1)$  symmetry. By the existence of this anomaly, the  $U_A(1)$  symmetry is regarded as explicitly broken at the quantum level. This mechanism plays an essential role on the resolution of the  $U_A(1)$  problem [17].

In this paper, we aim to reexamine the relation between QCD-monopoles and instantons through  $U_A(1)$  anomaly. By using the Monte Carlo simulation on  $SU(2)$  lattice, we measure the topological charge and the zero eigenvalues of the Dirac operator in both the “monopole dominating” and “monopole absent” background fields.

The Maximally Abelian (MA) gauge fixing [1] was advocated by 't Hooft in order to define the magnetic monopole in the renormalizable and the Lorentz invariant way in the continuum,  $(\partial_\mu \pm igA_\mu^3)A_\mu^\pm = 0$  where  $A_\mu^\pm = A_\mu^1 \pm iA_\mu^2$ . In the lattice formulation [4], this gauge fixing is expressed by diagonalizing the following operator  $X(n)$  through the gauge transformation  $U_\mu(n) \rightarrow V(n)U_\mu(n)V^\dagger(n + \hat{\mu})$ ,

$$X(n) = \sum_\mu \left\{ U_\mu(n) \sigma_3 U_\mu^\dagger(n) + U_\mu^\dagger(n - \hat{\mu}) \sigma_3 U_\mu(n - \hat{\mu}) \right\} \quad , \quad (2)$$

where  $U_\mu(n)$  are link variables. While  $X(n) = X^1(n)\sigma_1 + X^2(n)\sigma_2 + X^3(n)\sigma_3$ , this gauge fixing means that the off-diagonal elements are locally minimized in every sites by the gauge transformation, *i.e.*  $X^1(n) = X^2(n) = 0$ . Instead of this procedure, the gauge transformation is actually carried out by maximizing the gauge dependent variable  $R$  [4],

$$R = \sum_{n, \mu} \text{tr} \left\{ \sigma_3 U_\mu(n) \sigma_3 U_\mu^\dagger(n) \right\} . \quad (3)$$

Maximizing  $R$  is equivalent to making  $X(s)$  diagonal at all sites.

Once the gauge transformation is done by the above procedure, we factorize the SU(2) link variable  $U_\mu(n)$  into the abelian link variable  $u_\mu(n)$  and off-diagonal part  $M_\mu(n)$  [4] as

$$U_\mu(n) = M_\mu(n) \cdot u_\mu(n) , \quad (4)$$

where

$$u_\mu(n) \equiv \exp\{i\sigma_3\theta_\mu(n)\} , \quad (5)$$

$$M_\mu(n) \equiv \exp\{i\sigma_1 C_\mu^1(n) + i\sigma_2 C_\mu^2(n)\} . \quad (6)$$

Here,  $\theta_\mu(n)$  is the U(1) gauge field and  $C_\mu^1(n)$  and  $C_\mu^2(n)$  correspond to charged matter fields under a residual U(1) gauge transformation. Performing the U(1) gauge transformation on the original SU(2) link variable,  $u_\mu(n)$  and  $M_\mu(n)$  are transformed [4] as

$$u_\mu(n) \longrightarrow u'_\mu(n) = d(n)u_\mu(n)d^\dagger(n + \hat{\mu}) , \quad (7)$$

$$M_\mu(n) \longrightarrow M'_\mu(n) = d(n)M_\mu(n)d^\dagger(n) , \quad (8)$$

where  $d(n) = \exp\{i\sigma_3\varphi(n)\}$ . In this way,  $u_\mu(n)$  and  $M_\mu(n)$  behave like an abelian ‘photon’ and an adjoint ‘matter’ field, respectively.

Our next task is to look for the magnetic monopole in terms of the U(1) variables. We consider the product of U(1) link variables around an elementary plaquette,

$$u_{\mu\nu}(n) = u_\mu(n)u_\nu(n + \hat{\mu})u_\mu^\dagger(n + \hat{\nu})u_\nu^\dagger(n) = e^{i\sigma_3\theta_{\mu\nu}} , \quad (9)$$

where the abelian field strength  $\theta_{\mu\nu}(n) \equiv \theta_\nu(n + \hat{\mu}) - \theta_\nu(n) - \theta_\mu(n + \hat{\nu}) + \theta_\mu(n)$ . It should be noted that the U(1) plaquette variable is a multiple valued function as the abelian field strength due to the compactness of the residual U(1) gauge group. Then we can divide the abelian field strength into two parts as

$$\theta_{\mu\nu} = \bar{\theta}_{\mu\nu} + 2\pi N_{\mu\nu} , \quad (10)$$

where  $\bar{\theta}_{\mu\nu}$  is the regular part defined in  $-\pi < \bar{\theta}_{\mu\nu} \leq \pi$  and  $N_{\mu\nu} \in \mathbf{Z}$  is the modulo  $2\pi$  of  $\theta_{\mu\nu}$ . Here, it is known that the SU(2) link variable behaves as  $U_\mu \simeq u_\mu$  in the MA gauge [5]. In this sense,  $*N_{\mu\nu} \equiv \frac{1}{2}\varepsilon_{\mu\nu\rho\sigma}N_{\rho\sigma}$  corresponds to the Dirac string following the DeGrand-Toussaint's definition in the compact QED [18]. Then, monopole currents  $k_\mu(n)$  are identified as topological conserved currents defined by  $k_\mu(n) = \partial_\nu *N_{\mu\nu}(n + \hat{\mu})$  [18].

The next aim is to extract the contribution of the monopole dominated part from the abelian link variable. First, we define the two abelian gauge fields using two parts of the abelian field strength [19] as below,

$$\theta_\mu^{\text{Ph}}(n) \equiv \sum_m G(n-m) \partial_\lambda \bar{\theta}_{\lambda\mu}(m) , \quad (11)$$

$$\theta_\mu^{\text{Ds}}(n) \equiv 2\pi \sum_m G(n-m) \partial_\lambda N_{\lambda\mu}(m) , \quad (12)$$

where  $G(n-m)$  is the lattice Coulomb propagator.  $\theta_\mu^{\text{Ph}}$ , which is called ‘regular part’, is composed of the regular part of the abelian field strength  $\bar{\theta}_{\mu\nu}$  [19]. On the other hand,  $\theta_\mu^{\text{Ds}}$ , which is called ‘singular part’, is composed of the Dirac string part of the the abelian field strength [19]. It is noted that the sum of two values is the original abelian gauge field in the Landau gauge,  $\partial_\mu \theta_\mu^L(n) = 0$  [9],

$$\theta_\mu^{\text{Ph}}(n) + \theta_\mu^{\text{Ds}}(n) = \sum_m G(n-m) \partial_\lambda \theta_{\lambda\mu}(m) = \theta_\mu^L(n) , \quad (13)$$

where a superscript  $L$  denotes the Landau gauge. This procedure could correspond to dividing a ‘regular photon’ part from a ‘monopole’ part [19].

The corresponding SU(2) variables are reconstructed from  $\theta_\mu^{\text{Ph}}$  and  $\theta_\mu^{\text{Ds}}$  by multiplying the off-diagonal factor  $M_\mu$  [9]- [10] as

$$U_\mu^{\text{Ph}}(n) \equiv M_\mu(n) \exp\{i\sigma_3 \theta_\mu^{\text{Ph}}(n)\} , \quad (14)$$

$$U_\mu^{\text{Ds}}(n) \equiv M_\mu(n) \exp\{i\sigma_3 \theta_\mu^{\text{Ds}}(n)\} . \quad (15)$$

Here, the fact that  $U_\mu = U_\mu^{\text{Ph}} \cdot u_\mu^{\text{Ds}} = U_\mu^{\text{Ds}} \cdot u_\mu^{\text{Ph}}$  in the Landau gauge should be kept in mind. Thus, we shall use  $U_\mu^{\text{Ds}}$  as the ‘monopole dominating’ SU(2) link variable and  $U_\mu^{\text{Ph}}$  as the ‘monopole absent’ SU(2) link variable [9]- [10].

Next, we see how instantons are defined in the lattice formulation of QCD. Of course, in discretised space-time, we lose inherently the topology in the strict mathematical sense. Nevertheless, we expect that the topological character could be neatly identified near the continuum limit, since the variation of fields becomes smoother than the size of the lattice spacing [20]. We use the simplest expression for the topological charge [21] as

$$Q_L = \frac{1}{32\pi^2} \sum_n \varepsilon_{\mu\nu\rho\sigma} \text{Tr}\{U_{\mu\nu}(n)U_{\rho\sigma}(n)\} \quad , \quad (16)$$

where  $U_{\mu\nu}(n)$  is the plaquette variable. In the naive continuum limit,  $\text{Tr}\{U_{\mu\nu}(n)U_{\rho\sigma}(n)\}$  is reduced to  $\text{Tr}\{a^4 g^2 G_{\mu\nu} G_{\rho\sigma} + O(a^5)\}$  [21]. The value  $Q_L$  has not only  $O(a^2)$  corrections, but also renormalized multiplicative corrections of  $O(a^0)$  [22]. Consequently,  $Q_L$  is not an integer except for the continuum limit. However, it is known that the topological feature of  $Q_L$  can be extracted by removing the short wavelength fluctuations of the field configuration by using the following procedure. We use the cooling method [23], in which each link variable  $U_\mu$  is replaced by

$$\tilde{U}_\mu(n) = c \sum_{\mu \perp \nu} U_\nu(n) U_\mu(n + \hat{\nu}) U_\nu^\dagger(n + \hat{\mu}) \quad . \quad (17)$$

where a factor  $c$  ensures that a new link variable  $\tilde{U}_\mu$  is the element of SU(2) group. After this procedure, which is called a *cooling sweep*, the action is reduced;  $S[\tilde{U}_\mu] < S[U_\mu]$ . As a consequence, the field configuration locally smoothened. In Fig.1(a)-1(c), the cooling curves for the topological charge;  $Q_L$ , the integral of the absolute value of the topological density;  $I_Q^\dagger$  and the action divided by  $8\pi$ ;  $\tilde{S}$  are shown as typical examples.

In order to examine the eigenvalue of the Dirac operator on the lattice, we adopt the Wilson fermion operator [24];

$$\begin{aligned} D(n, m; U) \\ = \delta_{n, m} - \kappa \sum_\mu \left[ (r_w - \gamma_\mu) U_\mu(n) \delta_{n+\hat{\mu}, m} + (r_w + \gamma_\mu) U_\mu^\dagger(n - \hat{\mu}) \delta_{n-\hat{\mu}, m} \right] \quad . \end{aligned} \quad (18)$$

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<sup>†</sup>This quantity is defined as  $I_Q = \frac{1}{32\pi^2} \sum_n \varepsilon_{\mu\nu\rho\sigma} |\text{Tr}\{U_{\mu\nu}(n)U_{\rho\sigma}(n)\}|$  corresponding to the total number of topological pseudoparticles (instantons and anti-instantons).

where  $r_w$  is the Wilson parameter. We see that the choice  $r_w = 1$  is quite special since  $1 \pm \gamma_\mu$  are orthogonal projection operators. Thus, we use  $r_w = 1$  hereafter. In the naive argument, the Wilson fermion does not have the chiral symmetry due to the Wilson term. However, the effect of chiral symmetry breaking was systematically examined through the chiral Ward identities [25] and the Wilson term is necessary to maintain the axial vector anomaly [26]. It is known that in the strong coupling region the ordinary mass term and the Wilson term cancel out in the pseudo-scalar mass, which is symbolically called the pion mass, at some  $\kappa = \kappa_c(\beta)$ . In the strong coupling limit, the pion mass is zero at  $\kappa_c(\beta \rightarrow 0) \simeq \frac{1}{4}$  [27]. In the weak coupling regime, perturbative calculations indicate that the mass of the fermion becomes equal to zero along the line  $\kappa = \kappa_c(\beta)$ , which ends at  $\kappa_c(\beta \rightarrow \infty) \simeq \frac{1}{8}$ . In fact, many Monte Carlo simulations of lattice QCD with the Wilson fermion have shown the existence of such a critical line  $\kappa_c(\beta)$  in the  $\kappa - \beta$  plane, where the pion mass vanishes. The partial symmetry restoration would be realized near the vicinity of the critical line.

The Dirac operator  $D$  defined by the Wilson fermion loses a feature as the hermite operator owing to the discretization of the space-time. However, we can easily see that the operator  $\gamma_5 D$  or  $D^\dagger D$  is a hermite matrix. Then, we consider the eigenvalue problems for each operator by using the Lanczos algorithm. To discriminate quantum fluctuations among eigenvalues easily, we use the link variables smoothening the short wavelength with the cooling method in the same way as calculations of the topological charge [28]. The presence of chiral-asymmetric zero modes can be found by its sign change through the variation of the hopping parameter  $\kappa$  [29]. It is actually true that such a eigenvalue spectrum of  $\gamma_5 D$  inherently coincides with the operator  $D$ 's one at  $\kappa = \kappa_c$ , where the eigenvalue spectrum crosses a zero line.

We measure two sets of quantities, *i.e.* the eigenvalue spectrum of  $\gamma_5 D$  and the topological charge  $Q_L$ , by using ‘monopole dominating’ SU(2) link variable  $U_\mu^{\text{Ds}}$  and ‘photon dominating (monopole absent)’ SU(2) link variable  $U_\mu^{\text{Ph}}$  in the MA gauge on an  $8^4$  lattice with  $\beta = 2.4$ . Fig.2(a)-2(h) show the low-lying spectra of  $\gamma_5 D$  at various values of  $\kappa$  for

8 configurations with 20 cooling sweeps in the quenched approximation. It is clear that there exist chiral-asymmetric zero modes around  $\kappa \simeq 0.132$  in the background fields of the ‘monopole dominating’ part. Furthermore, all 8 configurations in the background fields dominated by monopoles hold an analogue of “the Atiyah-Singer index theorem”;  $n_+ - n_- \simeq Q_L$  where  $n_+(n_-)$  is the number of zero modes with positive (negative) “chirality” defined by the sign of its eigenvalue in the limit  $\kappa \uparrow \kappa_c$ . On the other hand, Fig.3(a)-3(h) show that no existence of chiral-asymmetric zero modes is found in background fields of the ‘photon dominating (monopole absent)’ part in all 8 configurations. In this case, each topological charge is also equal to zero [8]- [9]. By using 50 configurations, we also examine the eigenvalue of a positive-definite hermitian operator  $D^\dagger D$  associated with the Dirac operator  $D$ , in the ‘monopole dominating’ background fields and the ‘photon dominating’ background fields. There certainly exist almost zero modes in the ‘monopole dominating’ background, though we can not find the corresponding zero modes in the ‘photon dominating’ background. Therefore, in the ‘monopole dominating’ fields, the explicit breaking of the  $U_A(1)$  symmetry occurs due to the existence of the chiral-asymmetric zero modes.

In conclusion, we have investigated the eigenvalue problems for the Dirac operator, which is defined by the Wilson fermion, in the background fields of the ‘monopole dominating’ (Ds) part and the ‘photon dominating (monopole absent)’ (Ph) part by using the  $SU(2)$  lattice with  $8^4$  and  $\beta = 2.4$ . In only the background fields dominated by QCD-monopoles, the explicit breaking of the  $U_A(1)$  symmetry occurs due to the existence of the chiral-asymmetric zero modes. We have found the monopole dominance for the  $U_A(1)$  anomaly, and also confirmed that “the Atiyah-Singer index theorem” is satisfied in the backgrounds of QCD-monopoles.

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## REFERENCES

- [1] G. 't Hooft, Nucl. Phys. **B190** (1981) 455.
- [2] See, *e.g.* , R. Rajaraman, *Solitons and instantons* (North-Holland, Amsterdam, 1982).
- [3] G. 't Hooft, *in High energy physics*, ed. A. Zichichi (Editions Compositori, Bologna, 1975).  
S. Mandelstam, Phys. Rep. **C23** (1976) 245.
- [4] A. G. Kronfeld, G. Schierholz and U.-J. Wiese, Nucl. Phys. **B293** (1987) 461.
- [5] T. Suzuki and I. Yotsuyanagi, Phys. Rev. **D42** (1990) 4257.  
S. Hioki, S. Kitahara, Y. Matsubara, O. Miyamura, S. Ohno and T. Suzuki, Phys. Lett. **B272** (1991) 326.
- [6] S. Kitahara, Y. Matsubara and T. Suzuki, Prog. Theor. Phys. **93** (1995) 1.
- [7] H. Suganuma, H. Ichie, S. Sasaki and H. Toki, *Proc. of "Colour Confinement and Hadrons"*, (World Scientific, 1995), p.65.
- [8] H. Suganuma, K. Itakura and H. Toki, preprint (hep-th/9512141).
- [9] O. Miyamura and S. Origuchi, *Proc. of "Colour Confinement and Hadrons"*, (World Scientific, 1995), p.235.
- [10] H. Suganuma, A. Tanaka, S. Sasaki and O. Miyamura, Nucl. Phys. **B** (Proc. Suppl.) **47** (1996) 302.
- [11] M. N. Chernodub and F. V. Gubarev, JETP Lett. **62** (1995) 100.
- [12] R. C. Brower, K. N. Orginos and C.-I. Tan, Nucl. Phys. **B** (Proc. Suppl.) **53** (1997) 488; Phys. Rev. **D55** (1997) 6313.
- [13] S. Thurner, H. Markum and W. Sakuler, *Proc. of "Colour Confinement and Hadrons"*, (World Scientific, 1995), p.77 ;  
H. Markum, W. Sakuler and S. Thurner, Nucl. Phys. **B** (Proc. Suppl.) **47** (1996) 254;  
S. Thurner, M. Feurstein, H. Markum and W. Sakuler, Phys. Rev. **D54** (1996) 3457.
- [14] H. Suganuma, S. Sasaki, H. Ichie, F. Araki and O. Miyamura, Nucl. Phys. **B** (Proc. Suppl.) **53** (1997) 528.

- [15] A. Hart and M. Teper, Phys. Lett. **B371** (1996) 261.
- [16] S. Sasaki, M. Fukushima, A. Tanaka, H. Suganuma, H. Toki, O. Miyamura and D. Diakonov, *Proc. of "Quark Confinement and Hadron Spectrum II"*, (World Scientific, 1997), p.305 ;  
M. Fukushima, S. Sasaki, H. Suganuma, A. Tanaka, H. Toki and D. Diakonov, Nucl. Phys. **B** (Proc. Suppl.) **53** (1997) 494; Phys. Lett. **B399** (1997) 141.
- [17] G. 't Hooft, Phys. Rep. **142** (1986) 357.
- [18] T. A. DeGrand and D. Toussaint, Phys. Rev. **D22** (1980) 2478.
- [19] O. Miyamura, Phys. Lett. **B353** (1995) 91.
- [20] M. Lüscher, Commun. Math. Phys. **85** (1982) 39.
- [21] P. Di Vecchia, K. Fabricius, G. C. Rossi and G. Veneziano, Nucl. Phys. **B192** (1981) 392.
- [22] M. Campostrini, A. Di Giacomo and H. Panagopoulos, Phys. Lett. **212** (1988) 206.
- [23] E.-M. Ilgenfritz, M. L. Laursen, M. Müller-Preußker, G. Schierholz and H. Schiller, Nucl. Phys. **B268** (1986) 693.  
J. Hoek, M. Teper and J. Waterhouse, Nucl. Phys. **B288** (1987) 589.
- [24] K. G. Wilson, Phys. Rev. **D10** (1974) 2445.
- [25] M. Bochicchio, L. Maiani, G. Martinelli, G. Rossi and M. Testa, Nucl. Phys. **B262** (1985) 331.
- [26] L. H. Karsten and J. Smit, Nucl. Phys. **B183** (1981) 103.
- [27] N. Kawamoto, Nucl. Phys. **B190** (1981) 617.
- [28] I. Barbour and M. Teper, Phys. Lett. **B175** (1986) 445.
- [29] S. Itoh, Y. Iwasaki and T. Yoshié, Phys. Rev. **D36** (1987) 527.

## FIGURE CAPTIONS

**Fig.1** The cooling curves for  $Q_L$ ,  $I_Q$  and  $\tilde{S}$  are examined in the ‘monopole dominating’(Ds) part and the ‘photon dominating (monopole absent)’(Ph) part on an  $8^4$  lattice with  $\beta = 2.4$ . We show typical examples in the case of (a)  $Q_L(\text{Ds}) \neq 0$ , (b)  $Q_L(\text{Ds}) = 0$  and (c)  $Q_L(\text{Ph}) = 0$ .  $I_Q(\text{Ds})$  tends to remain finite during the cooling process. On the other hand,  $I_Q(\text{Ph})$  quickly vanish by only less than 5 cooling sweeps. Therefore, topological pseudoparticles seem unable to live in the Ph part, but only survive in the Ds part in the abelian gauge.

**Fig.2** The eigenvalue spectrum of  $\gamma_5 D$  in the ‘monopole dominating’ background fields (Ds part) as a function of the hopping parameter  $\kappa$  at 20 cooling sweeps on an  $8^4$  lattice with  $\beta = 2.4$ . The chiral-asymmetric zero modes are found in several configurations, where the topological charges have nonzero value.

**Fig.3** The eigenvalue spectrum of  $\gamma_5 D$  in the ‘photon dominating (monopole absent)’ background fields (Ph part) as a function of the hopping parameter  $\kappa$  at 20 cooling sweeps on an  $8^4$  lattice with  $\beta = 2.4$ . Both the chiral-asymmetric zero modes and the nonzero topological charges are not found in each configuration.

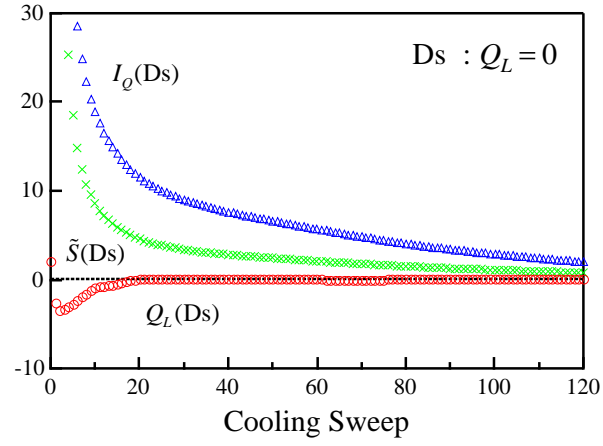


Fig. 1(a)

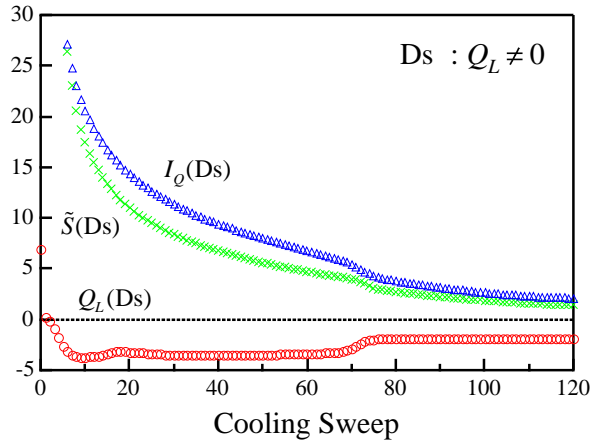


Fig. 1(b)

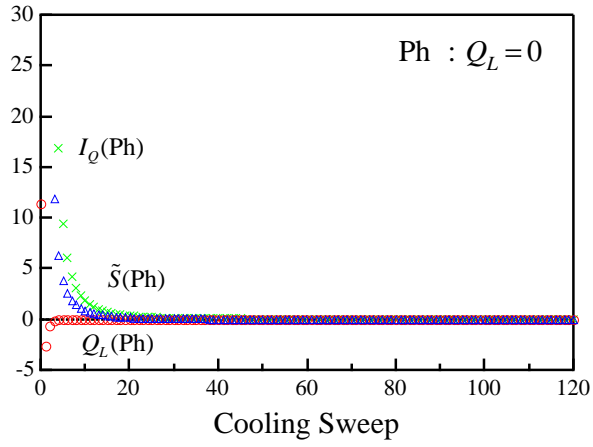


Fig. 1(c)

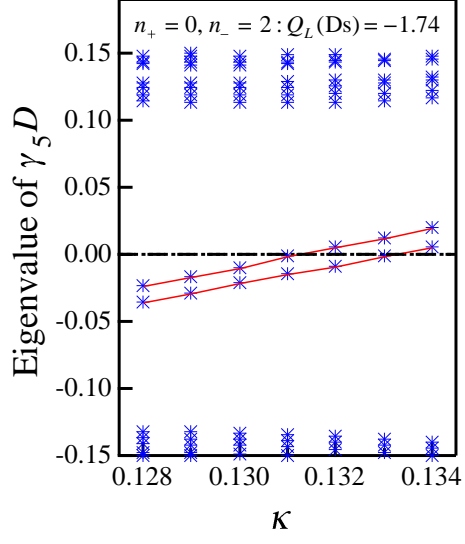


Fig. 2(a)

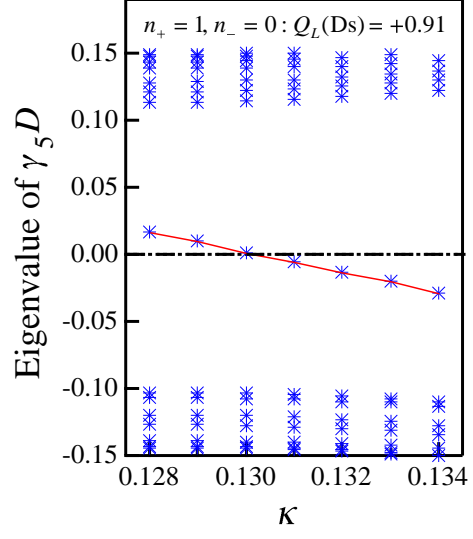


Fig. 2(b)

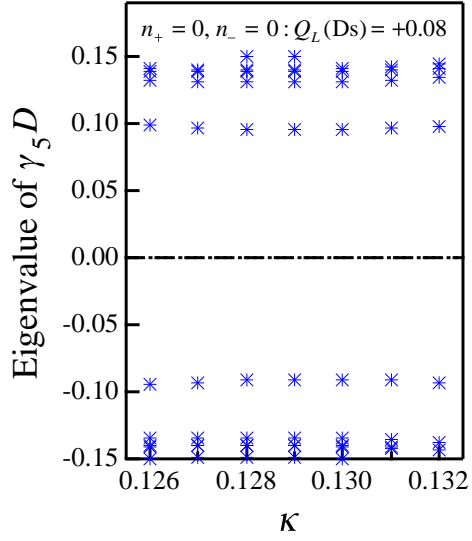


Fig. 2(c)

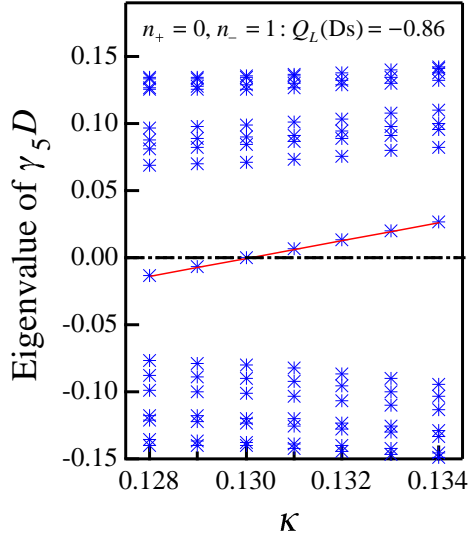


Fig. 2(d)

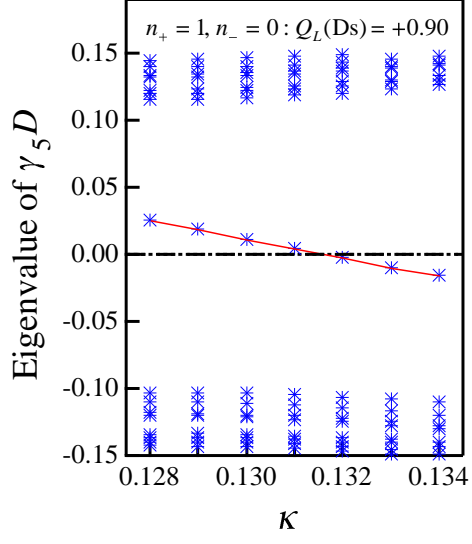


Fig. 2(e)

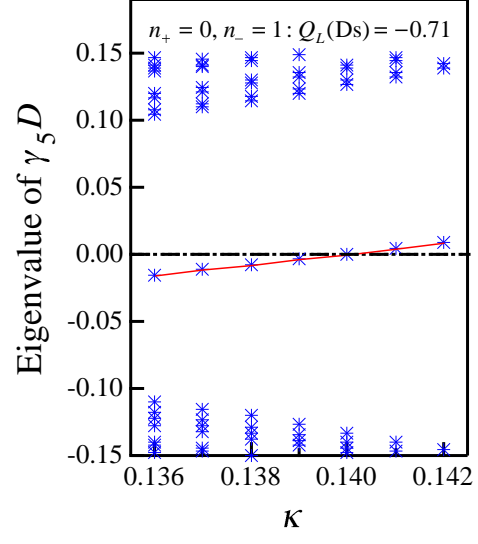


Fig. 2(f)

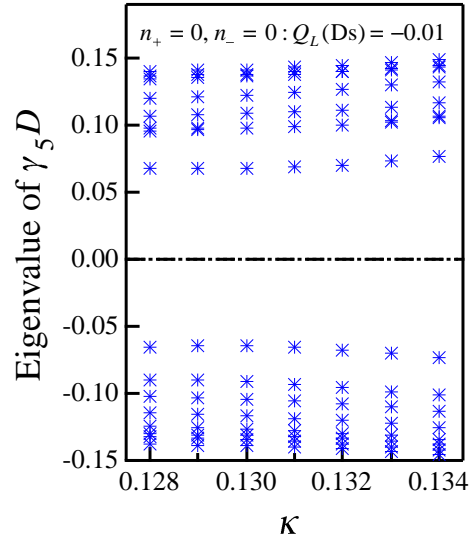


Fig. 2(g)

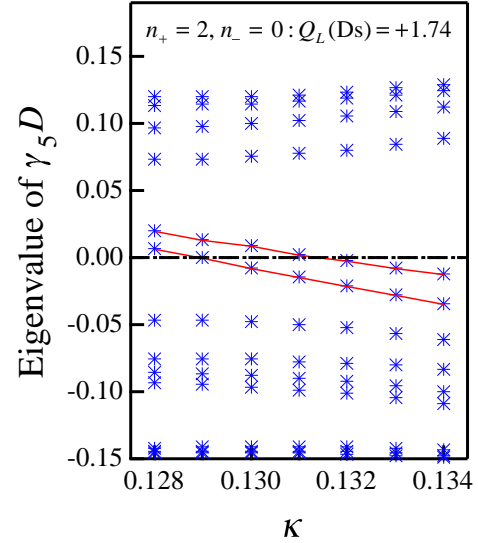


Fig. 2(h)

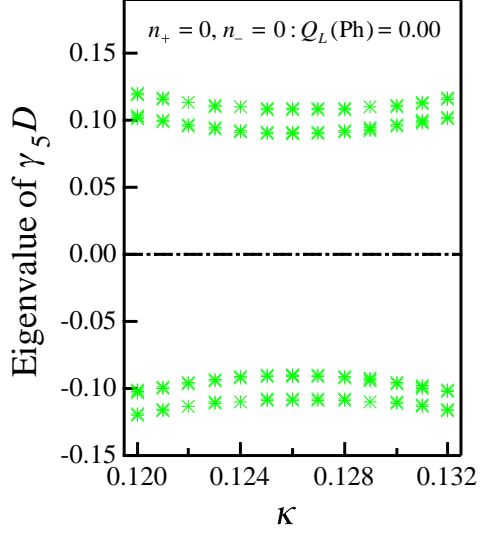


Fig. 3(a)

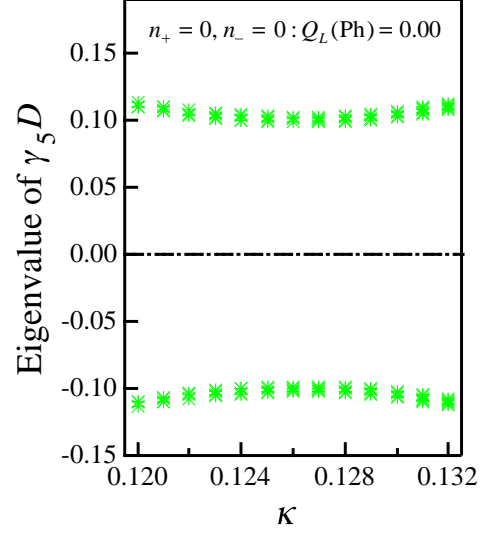


Fig. 3(b)

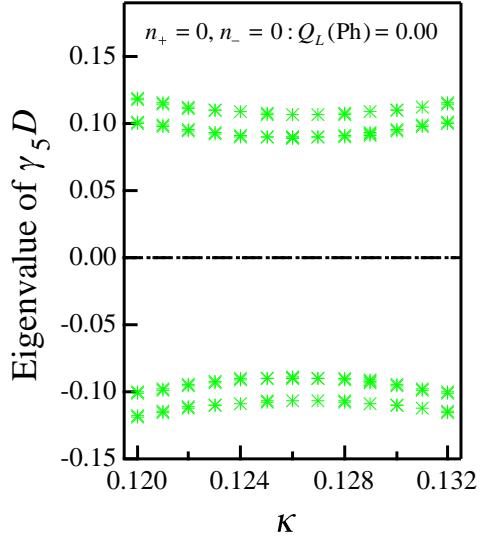


Fig. 3(c)

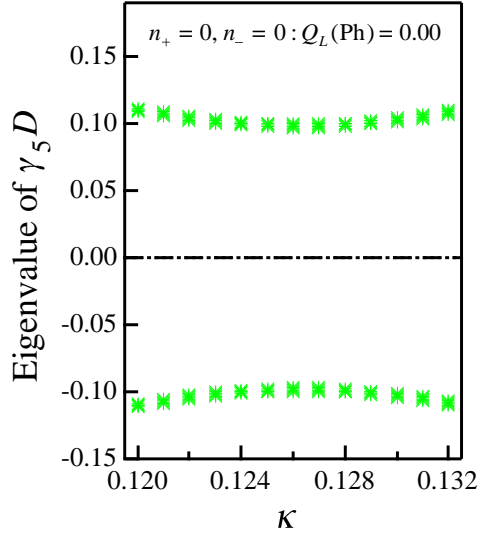


Fig. 3(d)



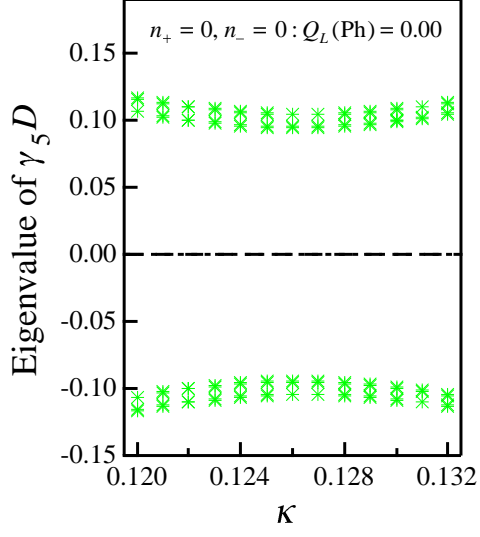


Fig. 3(e)

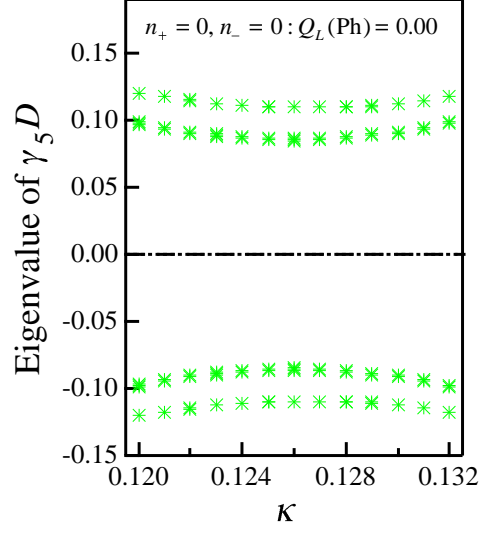


Fig. 3(f)

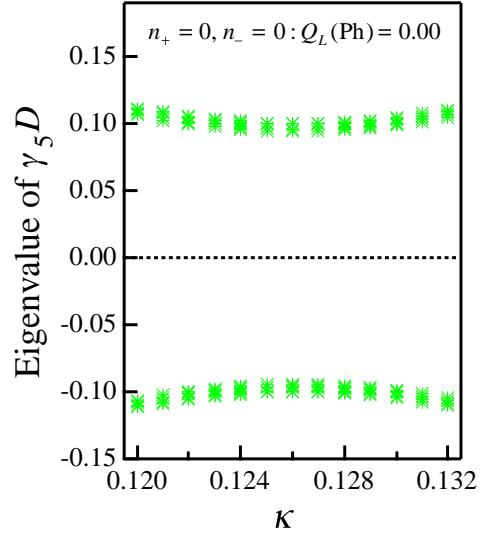


Fig. 3(g)

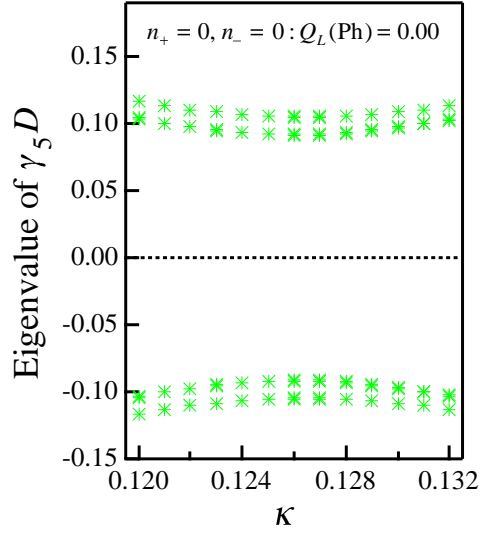


Fig. 3(h)